

[This question paper contains 8 printed pages.]

17/12/24

Your Roll No.....

Sr. No. of Question Paper : 1083

I

Unique Paper Code : 2352012301

Name of the Paper : Group Theory

Name of the Course : **B.Sc. (H) Mathematics**
UGCF

Semester : III – DSCC-7

Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **all** questions by selecting **two** parts from each question.
3. **All** questions carry equal marks.
4. Use of Calculator not allowed.

1. (a) Prove that the order of a permutation of a finite set written in disjoint cycle form is the least common multiple of the lengths of the cycles. (7.5)

(b) (i) Let S_n denote the symmetric group of degree n . In S_3 , find elements α and β such that $|\alpha| = 2$, $|\beta| = 2$ and $|\alpha\beta| = 3$.

(ii) Let $\beta \in S_7$ and $\beta^4 = (2\ 1\ 4\ 3\ 5\ 6\ 7)$. Then find β . (3,4.5)

(c) (i) Give two reasons to show that the set of odd permutations in S_n is not a subgroup of S_n .

(ii) Define even and odd permutations and show that the set of even permutations in S_n is a subgroup of S_n . (3, 4.5)

2. (a) (i) Let a be an element in a group G such that

$|a| = 15$. Find all left cosets of $\langle a^5 \rangle$ in $\langle a \rangle$.

- (ii) State and prove Lagrange's theorem.

(3,4.5)

- (b) Suppose that G is a group with more than one element and G has no proper, non-trivial subgroups.

Prove that $|G|$ is prime. (7.5)

- (c) Let $\mathbb{C}^*$ be the group of non-zero complex numbers under multiplication and let $H = \{a + bi \in \mathbb{C}^* \mid a^2 + b^2 = 1\}$. Give a geometrical description of the coset $(3 + 4i)H$. Give a geometrical description of the coset $(c + di)H$. (7.5)

3. (a) (i) Let G be a group and H be its subgroup.
Prove that if H has index 2 in G , then H is normal in G .
- (ii) If a group G has a unique subgroup H of some finite order, then show that H is normal in G . (3,4.5)
- (b) (i) Prove that a factor group of a cyclic group is cyclic. Is converse true? Justify your answer.
- (ii) Let G be a group and let $Z(G)$ be the center of G . If $G/Z(G)$ is cyclic, then show that G is Abelian. (3,4.5)
- (c) (i) Let ϕ be a group homomorphism from group G_1 to group G_2 and H be a subgroup of G_1 . Show that if H is cyclic, then $\phi(H)$ is cyclic.

(ii) How many homomorphisms are there from $\mathbb{Z}_{20}$ to $\mathbb{Z}_8$? How many are there onto $\mathbb{Z}_8$?

(3,4.5)

4. (a) (i) Suppose that ϕ is a homomorphism from $U(30)$ to $U(30)$ and $\text{Ker } \phi = \{1, 11\}$. If $\phi(7) = 7$, find all the elements of $U(30)$ that map to 7.

(ii) Let ϕ be a homomorphism from a group G_1 to group G_2 . Show that $\phi(a) = \phi(b)$ iff $a\text{Ker } \phi = b\text{Ker } \phi$.

(3,4.5)

(b) (i) Is $U(8)$ isomorphic to $U(10)$? Justify your answer.

(ii) Show that any infinite cyclic group is isomorphic to the group of integers under addition.

(3,4.5)

(c) If ϕ is an onto homomorphism from group G_1 to group G_2 , then prove that $G_1/\text{Ker } \phi$ is isomorphic to G_2 . Hence show that if G_1 is finite, then order of G_2 divides the order of G_1 . (7.5)

5. (a) Let G be a group and let $a \in G$. Define the inner automorphism of G induced by a . Show that the set of all inner automorphisms of a group G , denoted by $\text{Inn}(G)$, forms a subgroup of $\text{Aut}(G)$, the group of all automorphisms of G . Find $\text{Inn}(D_4)$. (7.5)

(b) Prove that the order of an element in a direct product of a finite number of finite groups is the lcm of the orders of the components of the element, i.e., $|(g_1, g_2, \dots, g_n)| = \text{lcm}(|g_1|, |g_2|, \dots, |g_n|)$. Also, find the number of elements of order 7 in $\mathbb{Z}_{49} \oplus \mathbb{Z}_7$. (7.5)

(c) Without doing any calculations in $\text{Aut}(\mathbb{Z}_{105})$, determine how many elements of $\text{Aut}(\mathbb{Z}_{105})$ have order 6. (7.5)

6. (a) For any group G , prove that $G/Z(G) \cong \text{Inn}(G)$. (7.5)

(b) Define the internal direct product of a collection of subgroups of a group G . Let $\mathbb{R}^*$ denote the group of all nonzero real numbers under multiplication. Let $\mathbb{R}^+$ denote the group of all positive real numbers under multiplication. Prove that $\mathbb{R}^*$ is the internal direct product of $\mathbb{R}^+$ and the subgroup $\{1, -1\}$. (7.5)

- (c) The set $G = \{1, 4, 11, 14, 16, 19, 26, 29, 31, 34, 41, 44\}$ is a group under multiplication modulo 45. Write G as an external and an internal direct product of cyclic groups of prime-power order.

(7.5)

Sr. No. of Question Paper : 1140
Unique Paper Code : 2352012302
Name of the Paper : Riemann Integration
Type of the Paper : DSCC-8
Semester : III

Programme: B.Sc. (H) Mathematics UGCF-2022
Duration : 3 Hours

Maximum Marks : 90

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt all questions by selecting three parts from each question.
3. All questions carry equal marks.
4. Use of Calculator is not allowed.

1. (a) Find the upper and lower Darboux integrals for $f(x) = x^2$ on the interval $[0, b]$ and show that $\int_0^b x^2 = \frac{b^3}{3}$.

(b) Let f be a bounded function on $[a, b]$. If P and Q are partitions of $[a, b]$ and $P \subseteq Q$, then prove that

$$L(f, P) \leq L(f, Q) \leq U(f, Q) \leq U(f, P)$$

(c) Let $f: [a, b] \rightarrow \mathbf{R}$ be a bounded function on $[a, b]$. Prove that if f is integrable on $[a, b]$, then for each $\epsilon > 0$, there exists a partition P of $[a, b]$ such that

$$U(f, P) - L(f, P) < \epsilon$$

(d) Let $f(x) = 2x + 1$ over the interval $[0, 2]$. Let $P = \left\{0, \frac{1}{2}, 1, \frac{3}{2}, 2\right\}$ be a partition of $[0, 2]$. Compute $U(f, P)$, $L(f, P)$ and $U(f, P) - L(f, P)$.

2. (a) Let f be an integrable function on $[a, b]$. Show that $-f$ is integrable on $[a, b]$ and

$$\int_a^b (-f) = - \int_a^b f$$

(b) Let $f: [0, 2] \rightarrow \mathbf{R}$ be defined as

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ -1, & \text{if } x \text{ is irrational} \end{cases}$$

Calculate the upper and lower Darboux Integrals for f on the interval $[0, 2]$. Is f integrable on $[0, 2]$?

(c) Let $f: [a, b] \rightarrow \mathbf{R}$ be a bounded function. Show that if f is integrable (Darboux) on $[a, b]$, then it is Riemann integrable on $[a, b]$.

(d) For a bounded function f on $[a, b]$, define the Riemann Sum associated with a partition P . Hence, give Riemann's definition of integrability.

3. (a) Prove that every bounded piecewise monotonic function f on $[a, b]$ is integrable.

(b) Show that if a function f is integrable on $[a, b]$, then $|f|$ is integrable on $[a, b]$ and $\left| \int_a^b f \right| \leq \int_a^b |f|$.

(c) If f is a continuous, non-negative function on $[a, b]$ and if $\int_a^b f = 0$, then prove that f is identically 0 on $[a, b]$. Give an example of a discontinuous non-zero function f on $[0, 1]$ for which $\int_0^1 f = 0$.

(d) State and prove Fundamental Theorem of Calculus I

4. (a) If u and v are continuous functions on $[a, b]$ that are differentiable on (a, b) , and u' and v' are integrable, prove that $\int_a^b uv' + \int_a^b u'v = u(b)v(b) - u(a)v(a)$. Hence evaluate $\int_0^\pi x \cos x$.

(b) Use the Fundamental Theorem of Calculus to calculate $\lim_{x \rightarrow 0} \frac{1}{x} \int_0^x e^{t^2} dt$.

(c) Let f be an integrable function on $[a, b]$. For x in $[a, b]$, let $F(x) = \int_a^x f(t) dt$. Then show that F is

uniformly continuous on $[a, b]$. For $f(x) = \begin{cases} 0, & t < 0 \\ t, & 0 \leq t \leq 1, \\ 4, & t > 1 \end{cases}$

(i) Determine the function $F(x) = \int_0^x f(t) dt$

(ii) Where is F continuous?

(d) For $t \in [0, 1]$, define $F(t) = \begin{cases} 0, & t < \frac{1}{2} \\ 1, & t \geq \frac{1}{2} \end{cases}$ and let $f(x) = x^2, x \in [0, 1]$. Show that f is

F -integrable and that $\int_0^1 f dF = f\left(\frac{1}{2}\right)$.

5. (a) Find the volume of the solid generated when the region enclosed by the curves $x = \sqrt{y}$ and $x = y/4$ is revolved about the x -axis.

(b) Use cylindrical shells to find the volume of the solid generated when the region under $y = x^2$ is revolved about the line $y = -1$.

(c) Find the exact arc length of the curve $x = \frac{1}{3}(y^2 + 2)^{3/2}$ from $y = 0$ to $y = 1$.

(d) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y -axis.

6. (a) Discuss the convergence or divergence of the following improper integrals:

(i) $\int_0^1 \frac{1}{\sqrt{x}} dx$ (ii) $\int_{-\infty}^{+\infty} e^x dx$

(b) Find the value of r for which the integral $\int_0^1 x^{-r} dx$ exists or converges, and determine the value of the integral.

(c) Show that the improper integral $\int_1^\infty \frac{\sin x}{x^2} dx$ converges absolutely.

(d) Define the Gamma function $\Gamma(m)$. Prove that $\Gamma(m)$ converges if $m > 0$.



02/01/2025

[This question paper contains 8 printed pages]

Your Roll No. :

Sl. No. of Q. Paper : **1313** **I**

Unique Paper Code : 2353012002

Name of the Paper : Mathematical Python
DSE

Name of the Course : **B.Sc.(Hons.)
Mathematics**

Semester : ☒ III

Time : 3 Hours

Maximum Marks : 90

Instructions for Candidates :

- Write your Roll No. on the top immediately on receipt of this question paper.
- Attempt **all** questions by selecting any **six** parts from question number **1**, and any **three** parts from question numbers **2** to **6** respectively.
- All** commands to be written in Python code.

1. Attempt any **6** parts. $2.5 \times 6 = 15$

- How do you find the mean and standard deviation of elements in a NumPy array ?

P.T.O.

- (b) Consider the list, list 1 = [1, 2, 4, 5, 7, 8, 3]. Write python code to reverse the elements of the list 1. Also find the length of the list 1.
- (c) How do you identify the data type of elements in a NumPy array ?

- (d) Consider an array, array 1 = $\begin{pmatrix} 2 & 3 \\ 8 & -6 \\ -3 & 4 \end{pmatrix}$, extract

an array $\begin{pmatrix} 8 & -6 \\ -3 & 4 \end{pmatrix}$ from array 1.

- (e) What is range function in NumPy ? Use it to generate first five natural numbers.
- (f) Use pretty printing to print the following expression.

$$\frac{d(x^2 + 2x + 3)}{dx}$$

- (g) Write a SymPy code to find the indefinite integral of the following function with respect to x.

$$f(x) = \sin x + \cos x$$

- (h) Write a SymPy code to find the limit of following function :

$$\lim_{x \rightarrow 0} \frac{\sin x}{x}$$

2. Attempt any **3** parts.

5×3=15

- (a) Using turtle graphics, write a python code that draw 100 squares turning right 10 degrees after each square complete.
- (b) Write a python code to plot the functions $y_1 = \sin x, y_2 = \sin 2x$ in a single plot. The code must include the following requirements :
- (i) Legends, grid and a title of the plot ;
 - (ii) Different colour for each function.
- (c) Consider an unbiased coin where the probability of getting head is $2/3$ and that of tail is $1/3$ in a toss. Write a python code to simulate such a coin toss.
- (d) Given a 2D array :

$a = [[1, 2, 3], [4, 5, 6], [7, 8, 9]]$

Use NumPy to

- (i) Extract the diagonal elements.
- (ii) Calculate the sum of each row.
- (iii) Transpose the array.

3. Attempt any **3** parts.

5×3=15

- (a) Write a python code using NumPy to find the characteristic polynomial, eigenvalues and eigenvectors for the given matrix A

$$A = \begin{pmatrix} 1 & 3 & 0 \\ 1 & -1 & 0 \\ 2 & 2 & 2 \end{pmatrix}$$

- (b) Use lambda to evaluate the following function at $x = 0$.

$$g(x) = \sqrt{\frac{2x^2 + 1}{3x - 5}}$$

- (c) Use SymPy to write the following expression and later to evaluate the same.

$$\int (\log(x))^2 dx$$

- (d) Use SymPy to print the first order and second order derivatives of the following function.

Also evaluate $f'(0)$ and $f''(1)$ for $a = 1$.

$$f(x) = 2\sin(\pi x) + 5e^{ax} + x^2$$

4. Attempt any **3** parts.

5×3=15

- (a) Create a 2 by 2 grid of subplots to plot the following 4 functions :

$$y_1 = x$$

$$y_2 = x^2$$

$$y_3 = x^3$$

$$y_4 = (1 + x)^4$$

where $x = \text{np.linspace}(0, 4, 200)$.

- (b) What output will the following code produce :

```
import numpy as np
```

```
arr1 = np.array([[ 1, -1, 3], [4, -3, 6]])
```

```
arr2 = np.array([[7, 0, 9], [5, 1, 10]])
```

```
print(np.concatenate(arr1, arr2], axis=0))
```

```
print(np.concatenate([arr1, arr2], axis=1))
```

- (c) Write a SymPy code to find the critical points of the following function and print the points of local maxima and minima.

$$f(x) = x^2 - 3x^{\frac{2}{3}}$$

- (d) Write a SymPy code to solve the following second order differential equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0$$

with the given Initial condition, $y(0) = 0$.

5. Attempt any **3** parts.

(a) Write a SymPy code to factorize the expression $x^3 - 6x^2 + 11x - 6$ and to expand $(x + y)^3$.

(b) Write a python code for the following :

(i) Using SymPy, find the series expansion of $\sin(x)$ around $x = \frac{\pi}{4}$, up to the order

$$\left(x - \frac{\pi}{4}\right)^3.$$

(ii) Define a recursive sequence where each term depends on the previous two terms :

$$a_n = a_{n-1} + 2 \times a_{n-2}$$

with initial terms $a_0 = 1$ and $a_1 = 2$.

(c) Write a NumPy code to solve the following system of linear equations using the inverse of coefficient matrix :

$$2x - 5y + 3z = 8$$

$$3x - y + 4z = 7$$

$$x + 3y + 2z = -3$$

(d) Write a SymPy code to find the following limit :

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

6. Attempt any **3** parts.

5×3=15

(a) Write a SymPy code to obtain and verify the solution of the following system of linear equations :

$$2x + 3y = 7$$

$$4x - y = 5$$

(b) For the given matrix :

$$A = \begin{pmatrix} 3 & -2 & 4 & -2 \\ 5 & 3 & -3 & -2 \\ 5 & -2 & 2 & -2 \\ 5 & -2 & -3 & 3 \end{pmatrix}$$

Write a python code using SymPy to perform the following operations :

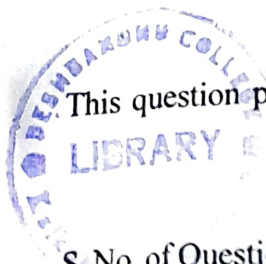
- (i) Create the matrix A and find its order.
- (ii) Find the eigenvalues and eigenvectors of the matrix A.
- (iii) Find the determinant and transpose of the matrix A.

- (c) Write a python code using pandas to create a data frame with the following data by reading as CSV file with the following columns : Name, Age and City.

Player	Age	Runs
A	25	80
B	24	110
C	29	145
D	21	62

- (i) Display the first row of the data frame.
 - (ii) Add a new column called Innings to the data frame with the following values : A : 30, B : 45, C : 42 and D : 17.
 - (iii) Sort the data frame by Age in the ascending order.
- (d) Write a python code using PyLaTeX to create a simple LaTeX document with a title, section and a single inline math expression for the area of the circle.

$$A = \pi r^2$$



This question paper contains 3 printed pages]

02/01/2025

Roll No.

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S.No. of Question Paper : 1314

Unique Paper Code : 2353012003

Name of the Paper : Number Theory

Type of the Paper : DSE-1

Semester : III

Name of the Course : B.Sc. (Hons) Mathematics

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting *two* parts from each question.

All questions carry equal marks.

Use of calculator is not allowed.

1. (a) A farmer purchased 100 head of livestock for a total cost of \$4000. Prices were as follow : calves \$120 each; lambs \$50 each; piglets \$25 each. If the farmer obtained at least one animal of each type, how many of each did he buy ? 7.5
- (b) Define a complete set of residues. If $a_1, a_2, \dots, a_n$ is a complete ste of residues modulo n and $\gcd(a, n) = 1$, prove that $aa_1, aa_2, \dots, aa_n$ is also a complete set of residues modulo n . 7.5
- (c) Solve the linear congruence $17x \equiv 3 \pmod{2.3.5.7}$. 7.5
2. (a) State and prove Wilson's Theorem. 7.5

P.T.O.

(b) (i) Find the unit digit of 3^{100} by the use of Fermat's theorem. 3.0

(ii) If $\gcd(a, 35) = 1$, show that $a^{12} \equiv 1 \pmod{35}$. 4.5

(c) Prove that if $n = p_1^{k_1} p_2^{k_2} \dots p_r^{k_r}$ is the prime factorization of $n > 1$, then the positive divisors of n are precisely those integers d of the form : 7.5

$$d = p_1^{a_1} p_2^{a_2} \dots p_r^{a_r} \text{ where } 0 \leq a_i \leq k_i \text{ (} i = 1, 2, \dots, r \text{)}.$$

3. (a) State and prove the Möbius inversion formula. 7.5

(b) If integer a have order k modulo n , then show that $a^h \equiv 1 \pmod{n}$ if and only if $k|h$. 7.5

(c) (i) Determine all primitive roots of 3^2 . 3.5

(ii) Determine the day of the week January 14, 2020. 4.0

4. (a) State and prove Euler's criterion. 7.5

(b) (i) Use Euler Theorem to show that for any integer a , $a^{37} \equiv a \pmod{1729}$. 4.0

(ii) Let r be a positive root of the odd prime p . Show that if $p \equiv 1 \pmod{4}$, then $-r$ is also a primitive root of odd primes. 3.5

(c) If $n \geq 1$ and $\gcd(a, n) = 1$, then show that $a^{\phi(n)} \equiv 1 \pmod{n}$. 7.5

5. (a) If $\gcd(m, n) = 1$ where $m > 2$, $n > 2$, then prove that mn has no primitive root. 7.5

(b) If $p \neq 3$ is an odd prime, then show that : 7.5

$$(3/p) = \{1 \text{ if } p \equiv \pm 1 \pmod{12} \text{ and } -1 \text{ if } p \equiv \pm 5 \pmod{12}\}.$$

- (c) The cipher text ALXWU VADCOJO has been enciphered with the Cipher :

$$C_1 \equiv 4P_1 + 11P_2 \pmod{26}$$

$$C_2 \equiv 3P_1 + 8P_2 \pmod{26}.$$

Derive the plaintext.

7.5

6. (a) Solve the quadratic congruence $3x^2 + 9x + 7 \equiv 0 \pmod{13}$. 7.5
- (b) Find the value of Legendre symbols $(18/43)$ and $(-72/131)$. 7.5
- (c) Using the linear cipher $C \equiv 5P + 11 \pmod{26}$, encrypt the message NUMBER THEORY. 7.5

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 5904

I

Unique Paper Code : 32351302

Name of the Paper : BMATH306 - Group Theory-1

Name of the Course : **B.Sc. Maths (Hons) (2019 onwards) LOCF**

Semester : III

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll. No. on the top immediately on receipt of this question paper.
2. Attempt all questions by selecting two parts from each question.
3. All parts of a question are to be attempted together.
4. Marks of the questions are indicated.
5. Use of Calculator not allowed.

1. (a) Let $G = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \in \mathbb{R} \sim \{0\} \right\}$. Show that G is an Abelian group.

P.T.O.

- (b) Let G be group and H be a non-empty subset of G . Prove that H is a subgroup of G if $ab^{-1} \in H, \forall a, b \in H$.

Hence prove that $H = \{A \in GL(2, \mathbb{R}) : \det A \text{ is a power of } 2\}$ is a subgroup of $GL(2, \mathbb{R})$.

- (c) Define a cyclic group. Show that every cyclic group is Abelian. Is the converse true? Justify.
(2×6=12)

2. (a) Prove that an Abelian group with two elements of order two must have a subgroup of order 4.

- (b) Describe the group of symmetries of an equilateral triangle.

- (c) Let G be a group and $a \in G$. If $|a| = n$, show that

$$\langle a \rangle = \{e, a, a^2, \dots, a^{n-1}\} \text{ \& } a^i = a^j \Leftrightarrow n \text{ divides } (i - j).$$

(2×6½=13)

3. (a) Let $G = \langle a \rangle$ be a cyclic group of order n . Then show that $G = \langle a^k \rangle \Leftrightarrow \gcd(k, n) = 1$.

- (b) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 1 & 3 & 5 & 4 & 7 & 6 & 8 \end{pmatrix}$ and

$$\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{pmatrix}$$

- (i) Write α and β as product of disjoint cycles.
- (ii) Compute $\alpha\beta$ and α^{-1} .
- (c) Suppose that a has order 15. Compute all the left cosets of $\langle a^5 \rangle$ in $\langle a \rangle$. (2x6=12)
4. (a) Define left coset of a subgroup H of a group G . Find all the left cosets of $H=\{1,11\}$ in $U(30)$.
- (b) (i) Prove that a subgroup H of a group G is normal in $G \Leftrightarrow xHx^{-1} \subseteq H, \forall x \in G$.
- (ii) Let H be a subgroup of a group G such that $x^2 \in H, \forall x \in G$. Show that H is a normal subgroup of G .
- (c) Let $\phi: G \rightarrow G'$ be a group homomorphism and let H be a subgroup of G . Prove that
- (i) if H is cyclic, then $\phi(H)$ is cyclic.
- (ii) if H is normal, then $\phi(H)$ is normal in $\phi(G)$. (2x6½=13)
5. (a) Let G be a finite group and H be a subgroup of G . Show that $|H|$ divides $|G|$.

(b) Determine all group homomorphisms from $\mathbb{Z}_n$ to $\mathbb{Z}_n$.

(c) Let H be a subgroup of G and $a, b \in G$. Prove that

$$(i) \quad aH = bH \Leftrightarrow a \in bH.$$

$$(ii) \quad aH = H \Leftrightarrow a \in H. \quad (2 \times 6 = 12)$$

6 (a) State and prove the First Fundamental Theorem of Group Homomorphisms.

(b) Let G be the symmetric group of permutations on the set $\{1, 2, 3, \dots, n\}$. For each σ in G , define

$$\text{Sgn}(\sigma) = \begin{cases} -1, & \text{if } \sigma \text{ is an odd permutation} \\ 1, & \text{if } \sigma \text{ is an even permutation.} \end{cases}$$

Prove that Sgn is a group homomorphism. Find the kernel of Sgn .

(c) (i) Let $\phi: G \rightarrow G^*$ be a group homomorphism. Prove that the $\text{Ker } \phi$ is a normal subgroup of the group G .

(ii) Show that $\phi: (\mathbb{R}, +) \rightarrow (\mathbb{R}^*, \cdot)$ defined as

$$\phi(x) = x^2 \text{ is not a group homomorphism.}$$

$$(2 \times 6\frac{1}{2} = 13)$$